

Indeterminate Forms:

- $\frac{0}{0}$
- $\frac{\pm\infty}{\pm\infty}$
- $\infty - \infty$ (or $-\infty + \infty$)
- $0 \cdot \infty$
- 0^0
- 1^∞
- ∞^0

Determinate Forms: (where a, b, c are arbitrary fixed real numbers, and we assume $a \neq 0, c \neq 1$)

- $\frac{a}{0}$
- $\frac{b}{\pm\infty}$ or $\frac{\pm\infty}{b}$
- $\infty + \infty$ (or $-\infty + -\infty$)
- $a \cdot \infty$
- $\pm\infty \cdot \pm\infty$
- 0^∞
- c^∞
- ∞^a or ∞^∞

For each of the problems 1-10, evaluate the limit or conclude that the function tends to $\infty, +\infty$, or $-\infty$.

1. $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 3}{x^3 + 4}$
2. $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 4}{x^2 - 2x + 3}$
3. $\lim_{x \rightarrow \infty} \frac{x^4 - 2x^2 + 6}{x^2 + 7}$
4. $\lim_{x \rightarrow +\infty} [x - \sqrt{x^2 - a^2}]$ where a is an arbitrary fixed real number
5. $\lim_{x \rightarrow -\infty} [x - \sqrt{x^2 - a^2}]$ where a is an arbitrary fixed real number
6. $\lim_{x \rightarrow 1^+} \frac{x-1}{\sqrt{x^2-1}}$
7. $\lim_{x \rightarrow 1^-} \frac{x-1}{\sqrt{x^2-1}}$
8. $\lim_{x \rightarrow +\infty} \frac{x^2+1}{x^{\frac{3}{2}}}$
9. $\lim_{x \rightarrow 2^-} \frac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$
10. $\lim_{x \rightarrow +\infty} [\sqrt{x^2 + 2x} - x]$
11. Suppose $f(x) \rightarrow +\infty$ and $g(x) \rightarrow -\infty$ as $x \rightarrow +\infty$. Find examples of functions f and g with these properties and such that:
 - a. $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = +\infty$
 - b. $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = -\infty$
 - c. $\lim_{x \rightarrow +\infty} [f(x) + g(x)] = A$, where A is an arbitrary real number

12. Suppose $f(x) \rightarrow \pm\infty$ and $g(x) \rightarrow \pm\infty$ as $x \rightarrow +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = +\infty$

b. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = -\infty$

c. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = A$, where A is an arbitrary real number

13. Suppose $f(x) \rightarrow 0$ and $g(x) \rightarrow 0$ as $x \rightarrow +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = +\infty$

b. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = -\infty$

c. $\lim_{x \rightarrow +\infty} \left[\frac{f(x)}{g(x)} \right] = A$, where A is an arbitrary real number

14. Suppose $f(x) \rightarrow 0$ and $g(x) \rightarrow \pm\infty$ as $x \rightarrow +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = +\infty$

b. $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = -\infty$

c. $\lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] = A$, where A is an arbitrary real number

Extra Credit: Suppose $f(x) \rightarrow 0$ and $g(x) \rightarrow 0$ as $x \rightarrow 0^+$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = 0$

b. $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = 1$

c. $\lim_{x \rightarrow 0^+} f(x)^{g(x)} = A$, where A is an arbitrary real number and $A \neq 0$ or 1