

Borough of Manhattan Community College
Calculus I
Applications of Derivatives and Curve Sketching

Problem #1

The equation of the graph of Folium of Descartes is given by $x^3 + y^3 - 6xy = 0$. Find the equation of the tangent line at the point $(\frac{4}{3}, \frac{8}{3})$ and sketch their graphs on the same set of axes.

Problem #2

The pressure p and volume V of an ideal diatomic gas undergoing an adiabatic process satisfy the equation $p v^{\frac{7}{5}} = c$, where c is a constant. Suppose that $V=32$ cubic inches, that $p=20$ pounds per square inches, and that the gas is being compressed at a rate of 0.1 cubic inches per second. Find the rate at which the pressure is changing.

Problem #3

For the following functions: $f(x) = x^3 - 3x^2 - 9x + 4$, $g(x) = 3x^5 - 25x^3 + 60x - 36$, $h(x) = \frac{x^2 + 4}{4 - x^2}$,

$$k(x) = \frac{4}{1 + x^2}$$

- Find all critical points
- Find intervals where the function is increasing; decreasing
- Find all points of inflection; if any
- Find intervals where the function is concave up; concave down
- Determine whether each critical point is a local maximum point, a local minimum point, or neither.

Problem #4

A certain wholesale paint dealer, in buying and distributing x cases of paint per week, incurs the following expenses.

- fixed costs (rent, etc.) of \$1200 per week
- an expense of \$60 x per week representing the cost of the paint itself to the dealer
- a cost of $\$ \frac{x^2}{24}$ per week for storing the inventory, handling accounts, etc.

Sales can be maintained at a rate of x cases per week at a price of p dollars per case, where $x=2160-24p$. Finally, due to space and other limitations, the dealer's maximum level of operations is the distribution of 1000 cases per week. Determine the price p at which the dealer should sell each case in order to maximize the weekly profit of the business. Plot the graph of the profit function and indicate the point where the maximum profit occurs.