Polynomials

Defn: A <u>term</u> is a product of constants and/or variables.

Examples: 2, $-3xy^3$, $24x^2$

A <u>polynomial</u> is a <u>sum</u> of terms.

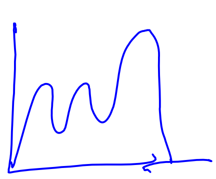
-2, $3x^2-9$, $4a^2b+2c+3c^4$, y^2-xz

$3x^2+-9$ $\quad$ y^2+-xz

<u>Why are we interested in polynomials?</u>

Polynomials can often be used to model real world phenomena:

Examples:

- The position or speed of a falling object
- Volume + area of rectangular solids (rectangles, cubes)
-  More complex equations or graphs can often be approximated by polynomials

What do we want to be able to do with polynomials?

- Simplify them
- perform operations such as addition/subtraction, multiplication (division – in a future lecture)
- factor them (next lecture)
- Solve equations containing polynomials (future lectures)

Simplifying Polynomials

Note: Subtraction can always be rewritten as addition with a negative number

$a - b = a + {}^-b$ or $a + (-b)$

Note: The order in which we add things does not matter.

$\Rightarrow$ We can rearrange terms in a polynomial into whatever order we want $a+b = b+a$

When can we combine two terms into a single term?

When the "variable part" is the same, we can combine the coefficients.

Examples: 1) $3x^2 - 5x^2 = 3x^2 + {}^-5x^2$

$= (3 + {}^-5)x^2$

$= \boxed{-2x^2}$

2) $4y^2 - 2y$ $\quad y^2 \neq y$

not "like terms"

The variable parts are different!

3) $\underbrace{4}_{}\underbrace{ab^2}_{} + \underbrace{2}_{}\underbrace{ab^2}_{} = (4+2)ab^2$

$= \boxed{6ab^2}$

4) $pq^2 - p^2q$

not "like terms"

$pq^2 \neq p^2q$

$p \cdot q \cdot q \neq p \cdot p \cdot q$

The variable parts are not the same!

Some more complex examples of "Like Terms" (Terms where the variable part is the same.):

5) $\underbrace{\underset{\text{coefficient}}{2}\underset{\text{variable part}}{(x-y)}} + \underbrace{\underset{\text{coefficient}}{x}\underset{\text{variable part}}{(x-y)}} = \boxed{(2+x)\underset{\text{variable part}}{(x-y)}}$

6) $\underset{\text{coefficient}}{a^2}\underset{\text{variable part}}{(b^2-1)} - \underset{\text{coefficient}}{b}\underset{\text{variable part}}{(b^2-1)} = a^2(b^2-1) + {}^-b(b^2-1)$

$= \boxed{\underset{\text{adding two coefficients}}{(a^2+{}^-b)}(b^2-1)}$

or $\boxed{(a^2-b)(b^2-1)}$

Simplifying Polynomials

Examples:

1) $3y^2 - 2xy + 1 - y^2 + 7 - 4xy$

$= 3y^2 + {}^{-}2xy + 1 + {}^{-}y^2 + 7 + {}^{-}4xy$

$= \underbrace{3y^2 + {}^{-}y^2} + \underbrace{{}^{-}2xy + {}^{-}4xy} + \underbrace{1 + 7}$

$= (3 + {}^{-}1)y^2 + ({}^{-}2 + {}^{-}4)xy + 8$

$= \boxed{{}^{-}2y^2 + {}^{-}6xy + 8}$ or $\boxed{{}^{-}2y^2 - 6xy + 8}$

Adding and Subtracting Polynomials

Examples!

1) $(x^2-4)+(2x^2+6x-1)$

$= \underbrace{(x^2+-4)+(2x^2+6x+-1)}$

all addition $\Rightarrow$ we can regroup and rearrange the terms.

$= x^2+-4+2x^2+6x+-1$

$= \underbrace{x^2+2x^2}+6x+\underbrace{-4+-1}$

$=(1+2)x^2+6x+-5$

$= \boxed{3x^2+6x+-5}$ or $\boxed{3x^2+6x-5}$

2) $(x^2-2x-6)-(2x^2+6x-9)$

$= (x^2+{}^{-}2x+{}^{-}6)+{}^{-}1(2x^2+6x+{}^{-}9)$

$= x^2+{}^{-}2x+{}^{-}6+{}^{-}2x^2+{}^{-}6x+9$

$= \underbrace{1x^2+{}^{-}2x^2}+\underbrace{{}^{-}2x+{}^{-}6x}+\underbrace{{}^{-}6+9}$

$= (1+{}^{-}2)x^2+({}^{-}2+{}^{-}6)x+3$

$= {}^{-}1x^2+{}^{-}8x+3$

$= \boxed{-x^2+{}^{-}8x+3}$ or $\boxed{-x^2-8x+3}$

3) $x-5[2x-(3-x)] = x+{}^{-}5[2x+{}^{-}1(3+{}^{-}x)]$

$= x+{}^{-}5[\underbrace{2x+{}^{-}3+x}]$

$= x+{}^{-}5[\underbrace{2x+1x}+{}^{-}3]$

$= x+{}^{-}5[(2+1)x+{}^{-}3]$

$= x+{}^{-}5\underbrace{[3x+{}^{-}3]}$

$= \underbrace{1x+{}^{-}15x}+15$

$= (1+{}^{-}15)x+15$

$= \boxed{-14x+15}$

Multiplying Polynomials

Distributive Property

For any values a, x and y:

$$a(x+y) = ax + ay$$

More generally:

For any values $a, x_1, x_2, \ldots, x_n$

$$a(x_1 + x_2 + x_3 + \ldots + x_n) = ax_1 + ax_2 + ax_3 + \ldots + ax_n$$

Examples:

1) $\underbrace{2x^2}_{\text{monomial (one term)}}(3x^2 + 6x - 1) = 2x^2(3x^2 + 6x + -1)$

$= 2x^2 \cdot 3x^2 + 2x^2 \cdot 6x + 2x^2 \cdot -1$

$= 2 \cdot 3 \cdot x^2 \cdot x^2 + 2 \cdot 6 \cdot x^2 \cdot x + 2 \cdot -1 \cdot x^2$

$= \boxed{6x^4 + 12x^3 + -2x^2}$

or $\boxed{6x^4 + 12x^3 - 2x^2}$

2) $\underbrace{(x+2)}_{\text{binomial (two terms)}}(3x-1) = (x+2)(3x + -1)$

$= (x+2) \cdot \underbrace{3x}_{\text{monomial}} + (x+2) \cdot \underbrace{-1}_{\text{monomial}}$

$= x \cdot 3x + 2 \cdot 3x + x \cdot -1 + 2 \cdot -1$

$= 3 \cdot x \cdot x + 2 \cdot 3 \cdot x + -1 \cdot x + -2$

$= 3x^2 + 6x + -1x + -2$

$= 3x^2 + (6 + -1)x + -2$

$= \boxed{3x^2 + 5x + -2}$ or $\boxed{3x^2 + 5x - 2}$

We ended up multiplying every term in the first polynomial by every term in the second polynomial.

3) $(x-2y)(x^2+2xy-3y^2) = (x+{}^-2y)(x^2+2xy+{}^-3y^2)$

$= (x+{}^-2y)x^2 + (x+{}^-2y)2xy + (x+{}^-2y){}^-3y^2$

$= x\cdot x^2 + {}^-2y\cdot x^2 + x\cdot 2xy + {}^-2y\cdot 2xy + x\cdot {}^-3y^2$
$+ {}^-2y\cdot {}^-3y^2$

$= x^3 + \underbrace{{}^-2x^2y + 2x^2y}_{0} + \underbrace{{}^-4xy^2 + {}^-3xy^2} + 6y^3$

$= x^3 + \underbrace{({}^-2+2)}_{0}x^2y + \underbrace{({}^-4+{}^-3)}_{-7}xy^2 + 6y^3$

$= \boxed{x^3 + {}^-7xy^2 + 6y^3}$

or

$\boxed{x^3 - 7xy^2 + 6y^3}$

General Pattern!

We must multiply **every term** in the first polynomial by **every term** in the second polynomial.

Careful! Be sure to change all subtraction to addition. Keep the negative sign with the correct term — the one that comes *after* the negative sign!