Lab #1: Limits
Calculus I, Prof. Wladis

Part I of this Lab: Do all the **a** and **b** parts below by the deadline for Part I. Leave room for the **c** parts to be added next week.

Part II of this Lab: Add in your answers to all the **c** parts below. Submit the whole file with all of your a, b, and c answers for the deadline for Part II.

1. Graph $y = x^{-2}$:
   a. Look at the graph: What do you think happens as x approaches zero from the left? When x approaches zero from the right? Describe this in a few sentences.
   b. Make a table of y values for each of the following x values.

x-value	y-value
1	
0.1	
0.01	
0.001	
0.0001	

Now make another table of values for each of the following x values.

x-value	y-value
-2	
-0.2	
-0.02	
-0.002	
-0.0002	

Describe in words what you think happens to y as x gets closer to zero from the right, and what happens when x gets closer to zero from the left, based on what you see in these tables of values.

   c. Evaluate the limits algebraically. If the limit increases or decreases without bound, write INF for ∞ and -INF for $-\infty$, respectively.
   Find $\lim_{x \to 0^-} x^{-2} = ?$
   Find $\lim_{x \to 0^+} x^{-2} = ?$

2. Graph $y = x^{-1}$:
   a. Look at the graph: What do you think happens as x approaches zero from the left? When x approaches zero from the right? Describe this in a few sentences.
   b. Make a table of y values for each of the following x values. Round to the nearest tenth.

x-value	y-value
3	

0.3	
0.03	
0.003	
0.0003	

Now make another table of values for each of the following x values.

x-value	y-value
-2	
-0.2	
-0.02	
-0.002	
-0.0002	

Describe in words what you think happens to y as x gets closer to zero from the right, and what happens when x gets closer to zero from the left, based on what you see in these tables of values.

c. Evaluate the limits algebraically. If the limit increases or decreases without bound, write INF for ∞ and -INF for $-\infty$, respectively.

Find $\lim\limits_{x\to 0^-} x^{-1} = ?$

Find $\lim\limits_{x\to 0^+} x^{-1} = ?$

3. Graph $y = \dfrac{2x}{\sqrt{x^2-1}}$:

a. Look at the graph: What do you think happens as x increases without bound?   When x decreases without bound? Describe this in a few sentences.

b. Make a table of y values for each of the following x values. Round to the nearest thousandth. If any values are undefined, write undefined.

x-value	y-value
1	
2	
4	
8	
16	

Now make another table of values for each of the following x values. Round to the nearest ten-thousandth. If any values are undefined, write undefined.

x-value	y-value
-5	
-10	
-20	
-40	
-80	

Describe in words what you think happens to y as x increases without bound, and what happens when x decreases without bound, based on what you see in these tables of values.

c. Evaluate the limits algebraically. If the limit increases or decreases without bound, write INF for ∞ and -INF for $-\infty$, respectively.

Find $\lim\limits_{x\to-\infty} \dfrac{2x}{\sqrt{x^2-1}}$ =?

Find $\lim\limits_{x\to\infty} \dfrac{2x}{\sqrt{x^2-1}}$ =?

The following questions cannot be evaluated algebraically yet, because we haven't learned how to find the limits of these kinds of functions yet. However, we can use both the visual graph and sample values to make educated guesses about the limits of these functions.

4. Graph $y = \dfrac{e^{2x}-1}{x}$:

a. Look at the graph, and try zooming in on the graph where x=0: What do you think happens as x approaches zero from the left and the right? Describe this in a few sentences.

b. Notice that if you try to look up the value of y when x=0, you may not be able to find a value for y: this is because this function is not defined when x=0.

Make a table of y values for each of the following x values. Round your answer to the nearest ten-thousandth.

x-value	y-value
1	
0.1	
0.01	
0.001	
0.0001	

Now make another table of values for each of the following x values. Round your answers to the nearest ten-thousandth.

x-value	y-value
-1	
-0.1	
-0.01	
-0.001	
-0.0001	

Describe in words what you think happens to y as x gets closer to zero from the right and from the left, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an **educated guess** about the following limit. If the limit increases or decreases without bound, write INF for ∞ and -INF for $-\infty$, respectively.

Find $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{x}$ =?

5. Graph $y = \dfrac{\ln x}{x}$:

a. Look at the graph: What do you think happens as x increases without bound? Describe this in a few sentences.

b. Make a table of y values for each of the following x values. Round your answers to the nearest ten-thousandth.

x-value	y-value
2	
20	
200	
2,000	
20,000	

Describe in words what you think happens to y as x increases without bound, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an **educated guess** about the following limit. If the limit increases or decreases without bound, write INF for ∞ and -INF for −∞, respectively.

Find $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = ?$

6. Graph $y = x^x$:

a. Look at the graph, and try zooming in on the graph where x=0: What do you think happens as x approaches zero from the right? Describe this in a few sentences.

b. Notice that if you try to look up the value of y when x=0, you may not be able to find a value: this is because this function is not defined when x=0.

Make a table of y values for each of the following x values. Round your answers to the nearest whole number.

x-value	y-value
2	
0.2	
0.02	
0.002	
0.0002	

Describe in words what you think happens to y as x gets closer to zero from the right, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an **educated guess** about the following limit. If the limit increases or decreases without bound, write INF for ∞ and -INF for −∞, respectively.

Find $\lim\limits_{x\to 0^+} x^x = ?$

7. Graph $y = \dfrac{e^x}{x}$:

a. Look at the graph: What do you think happens as x increases without bound? Describe this in a few sentences.

b. Make a table of y values for each of the following x values. Round to the nearest whole number.

x-value	y-value
2	
4	
8	
16	
32	

Describe in words what you think happens to y as x increases without bound, based on what you see in these tables of values.

c. Based on your answers to a) and b), make an **educated guess** about the following limit. If the limit increases or decreases without bound, write INF for ∞ and -INF for $-\infty$, respectively.

Find $\lim\limits_{x\to\infty} \dfrac{e^x}{x} = ?$

8. Graph $y = \sin\dfrac{1}{x}$:

Then set the Bounds to x_{min}=-1, x_{max}=1, y_{min}=-1, y_{max}=1.

a. Look at the graph: what do you think happens as x approaches zero?

b. Change the bounds to x_{min}=-0.1, x_{max}=0.1, y_{min}=-1, y_{max}=1, and look at the resulting graph. Repeat this with the bounds x_{min}=-0.01, x_{max}=0.01, y_{min}=-1, y_{max}=1, x_{min}=-0.001, x_{max}=0.001, y_{min}=-1, y_{max}=1, and x_{min}=-0.0001, x_{max}=0.0001, y_{min}=-1, y_{max}=1. Describe in words what you think the value of y does as x approaches zero.

c. What do you think happens to $f(x)$ as $x \to 0$? Do you think that the limit exists? Why or why not? (If it does exist, explain what the limit is.)