Wladis, BMCC/CUNY **Project 1: Limits** **Names:**____

Write out your answers here and hand in the paper in hard copy. The *Prover* should write down the answers to all initial questions, and the *Explainer* should write out all explanations (but all students should discuss all steps together). As usual, rotate through the rolls so that each student plays a different role for each problem. **Turn in one copy per group.**

1. Draw a function $f(x)$ where $\lim_{x\to -10} f(x) \neq f(-10)$:
(i.e. the limit as $x \to -10$ is different from the exact value of f at $x = -10$)

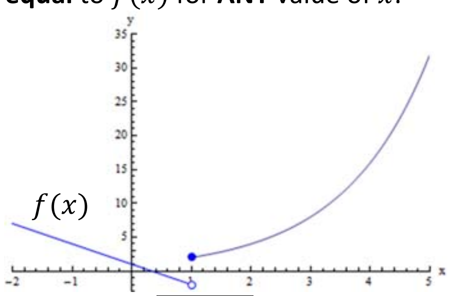
Draw graph here:

Explain how you know that $\lim_{x\to -10} f(x) \neq f(-10)$:

2. Answer questions about each of the following functions, given the graph:

a. $\lim_{x\to 1^-} f(x) =$ ☐ (see graph below)

b. On the graph below, draw another function $g(x)$ that has the **same limit** as $f(x)$ as $x \to 1^-$, but that is **NOT equal** to $f(x)$ for **ANY** value of x.

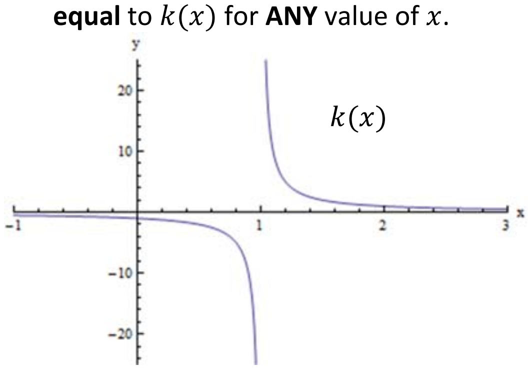


Explain how you know that $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} g(x)$:

Explain how you know that $f(1) \neq g(1)$:

c. $\lim_{x\to \infty} k(x) =$ ☐ (see graph below)

d. On the graph below, draw another function $g(x)$ that has the **same limit** as $k(x)$ as $x \to \infty$, but that is **NOT equal** to $k(x)$ for **ANY** value of x.



Explain how you know that $\lim_{x\to \infty} f(x) = \lim_{x\to \infty} g(x)$:

Explain how you know that $f(x) \neq g(x)$ for **any** value of x:

3. Draw two graphs, $f(x)$ and $g(x)$ so that:

a. $\lim_{x\to 3} f(x)$ does NOT exist

b. $\lim_{x\to \infty} g(x)$ does NOT exist

4. Consider the work that several different students have written:

A. As $x \to 3^+$, $\frac{1}{x-3} \to \frac{1}{3^+-3} \to \frac{1}{0^+} \to \infty$

B. When $x = 3$, $\frac{1}{x-3} = \frac{1}{3-3} = \frac{1}{0} = \infty$

C. $\lim\limits_{x \to 3^+} \frac{1}{x-3} = \frac{1}{3^+-3} = \frac{1}{0^+} = \infty$

D. When $x = 3$, $\frac{1}{x-3} \to \frac{1}{3-3} \to \frac{1}{0} \to \infty$

E. $\lim\limits_{x \to 3^+} \frac{1}{x-3} = \infty$ because $3^+ - 3 \to 0^+$ and $\frac{1}{0^+} \to +\infty$

a. Some of this work uses arrows and some of this work uses equals signs—explain what each of these symbols represents and why they are different from one another.

b. Look carefully at each of the student's answers, paying in particular attention to the arrows and equals signs. **Circle** every arrow or equals sign in the work above that is used incorrectly.
Then **explain why** each one is incorrect (if none are incorrect for a particular student, just write "none"):

A.

B.

C.

D.

E.

c. The 3^+ versus the 3 used in the students' work above represent two different ideas—**explain** what each of these represents and **how they are different** from one another.

d. Consider the ∞ sign—**explain what this sign means** in the context of limits. Is it ever appropriate to use an equals sign with the ∞ sign? If so, when? If not, why not?

e. Consider the following statements. First, for each of these statements, **explain what they are asking you to do** (do NOT simply give the answer—explain what each statement is *asking* instead).
Then identify which statements are asking the same thing by **circling and drawing arrows** between any pairs of statements that mean the same thing.

i. As $x \to 3^+$, $\frac{1}{x-3} \to \boxed{?}$

ii. $\lim\limits_{x \to 3} \frac{1}{x-3} = \boxed{?}$

iii. When $x = 3$, $\frac{1}{x-3} = \boxed{?}$

iv. As $x \to 3$, $\frac{1}{x-3} \to \boxed{?}$

v. $\lim\limits_{x \to 3^+} \frac{1}{x-3} = \boxed{?}$

5. Suppose $f(x) \to +\infty$ and $g(x) \to -\infty$ as $x \to +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x\to+\infty} [f(x) + g(x)] = +\infty$

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x\to+\infty} f(x) =$
$g(x) =$	$\lim_{x\to+\infty} g(x) =$
$f(x) + g(x) =$	$\lim_{x\to+\infty} [f(x) + g(x)] =$

b. $\lim_{x\to+\infty} [f(x) + g(x)] = -\infty$

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x\to+\infty} f(x) =$
$g(x) =$	$\lim_{x\to+\infty} g(x) =$
$f(x) + g(x) =$	$\lim_{x\to+\infty} [f(x) + g(x)] =$

c. $\lim_{x\to+\infty} [f(x) + g(x)] = A$, where A is any arbitrary real number that you choose yourself.

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x\to+\infty} f(x) =$
$g(x) =$	$\lim_{x\to+\infty} g(x) =$
$f(x) + g(x) =$	$\lim_{x\to+\infty} [f(x) + g(x)] =$

d. Think carefully about what the symbols ∞ and $-\infty$ represent. (Hint: Are they fixed values, or do they describe a particular *behavior*?) Now use your work from parts a-c to explain why $\infty + -\infty$ is an indeterminate form.

e. Thinking carefully again about the meaning of the symbols ∞ and $-\infty$, explain how this is different from the determinate forms $\infty + \infty$ and $-\infty + -\infty$.

6. Suppose $f(x) \to \pm\infty$ and $g(x) \to \pm\infty$ as $x \to +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = +\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} [\frac{f(x)}{g(x)}] =$

b. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = -\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} [\frac{f(x)}{g(x)}] =$

c. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = A$, where A is any arbitrary real number that you choose yourself.

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} [\frac{f(x)}{g(x)}] =$

d. Use your work from parts a-c to explain why $\frac{\pm\infty}{\pm\infty}$ is an indeterminate form.

e. Explain what happens when you divide any fixed number by a series of larger and larger numbers. How does the result change? Use this information to explain why the form $\frac{a}{\pm\infty}$ (where a is any fixed number) is a determinate form.

f. Using your answers to d and e, explain how the indeterminate form $\frac{\pm\infty}{\pm\infty}$ is different from the determinate form $\frac{a}{\pm\infty}$ (where a is any fixed number).

7. Suppose $f(x) \to 0$ and $g(x) \to 0$ as $x \to +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = +\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} [\frac{f(x)}{g(x)}] =$

b. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = -\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] =$

c. $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] = A$, where A is any arbitrary real number that you choose yourself.

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x \to +\infty} f(x) =$

$g(x) =$ $\lim_{x \to +\infty} g(x) =$

$\frac{f(x)}{g(x)} =$ $\lim_{x \to +\infty} \left[\frac{f(x)}{g(x)}\right] =$

d. Think carefully about what the symbol 0 represents in the statements $f(x) \to 0$ and $g(x) \to 0$. (Hint: Is it a fixed value, or does it describe a particular *behavior*?) Now use your work from parts a-c to explain why $\frac{0}{0}$ is an indeterminate form.

e. Explain what happens when you divide any fixed number by a series of smaller and smaller numbers that become closer and closer to zero. How does the result change? Use this information to explain why the form $\frac{a}{0}$ (where a is any fixed number) is a determinate form.

f. Thinking carefully again about the meaning of the symbol zero in the statements $f(x) \to 0$ and $g(x) \to 0$, explain how the indeterminate form $\frac{0}{0}$ is different from the determinate form $\frac{a}{0}$ (where $a \neq 0$).

8. Suppose $f(x) \to 0$ and $g(x) \to \pm\infty$ as $x \to +\infty$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x\to+\infty} [f(x) \cdot g(x)] = +\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x\to+\infty} f(x) =$

$g(x) =$ $\lim_{x\to+\infty} g(x) =$

$f(x) \cdot g(x) =$ $\lim_{x\to+\infty} [f(x) \cdot g(x)] =$

b. $\lim_{x\to+\infty} [f(x) \cdot g(x)] = -\infty$

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x\to+\infty} f(x) =$

$g(x) =$ $\lim_{x\to+\infty} g(x) =$

$f(x) \cdot g(x) =$ $\lim_{x\to+\infty} [f(x) \cdot g(x)] =$

c. $\lim_{x\to+\infty} [f(x) \cdot g(x)] = A$, where A is any arbitrary real number that you choose yourself.

Prover (choose the functions): *Explainer (check the limits):*

$f(x) =$ $\lim_{x\to+\infty} f(x) =$

$g(x) =$ $\lim_{x\to+\infty} g(x) =$

$f(x) \cdot g(x) =$ $\lim_{x\to+\infty} [f(x) \cdot g(x)] =$

d. Think carefully about what the symbols ∞, $-\infty$ and 0 represent in the statements $f(x) \to 0$ and $g(x) \to \pm\infty$. (Hint: Are they fixed values, or do they describe a particular *behavior*?) Now use your work from parts a-c to explain why $0 \cdot \pm\infty$ is an indeterminate form.

e. Thinking carefully again about the meaning of the symbols ∞ and $-\infty$, explain how this is different from the determinate forms $\pm\infty \cdot \pm\infty$ and $a \cdot \pm\infty$ (where $a \neq 0$).

Extra Credit: Suppose $f(x) \to 0$ and $g(x) \to 0$ as $x \to 0^+$. Find examples of functions f and g with these properties and such that:

a. $\lim_{x \to 0^+} f(x)^{g(x)} = 0$

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x \to +\infty} f(x) =$
$g(x) =$	$\lim_{x \to +\infty} g(x) =$
$f(x)^{g(x)} =$	$\lim_{x \to +\infty} [f(x)^{g(x)}] =$

b. $\lim_{x \to 0^+} f(x)^{g(x)} = 1$

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x \to +\infty} f(x) =$
$g(x) =$	$\lim_{x \to +\infty} g(x) =$
$f(x)^{g(x)} =$	$\lim_{x \to +\infty} [f(x)^{g(x)}] =$

c. $\lim_{x\to 0^+} f(x)^{g(x)} = A$, where A is any arbitrary real number except 0 and 1 that you choose yourself.

Prover (choose the functions):	*Explainer (check the limits):*
$f(x) =$	$\lim_{x\to +\infty} f(x) =$
$g(x) =$	$\lim_{x\to +\infty} g(x) =$
$f(x)^{g(x)} =$	$\lim_{x\to +\infty} [f(x)^{g(x)}] =$

d. Use your work from parts a-c to explain why 0^0 is an indeterminate form.

e. Explain how this is different from the determinate form $0^{\pm\infty}$ and the fixed numbers 0^a or a^0 (where $a \neq 0$).